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Vector quantization in DCT domain using fuzzy possibilistic c-means based on penalized and compensated constraints

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Abstract

In this paper, fuzzy possibilistic c-means (FPCM) approach based on penalized and compensated constraints are proposed to vector quantization (VQ) in discrete cosine transform (DCT) for image compression. These approaches are named penalized fuzzy possibilistic c-means (PFPCM) and compensated fuzzy possibilistic c-means (CFPCM). The main purpose is to modify the FPCM strategy with penalized or compensated constraints so that the cluster centroids can be updated with penalized or compensated terms iteratively in order to find near-global solution in optimal problem. The information transformed by DCT was separated into DC and AC coefficients. Then, the AC coefficients are trained by using the proposed methods to generate better codebook based on VQ. The compression performances using the proposed approaches are compared with FPCM and conventional VQ method. From the experimental results, the promising performances can be obtained using the proposed approaches.

Keywords: Discrete cosine transform; Fuzzy c-means; Fuzzy possibilistic c-means; Vector quantization

1. Introduction

Discrete cosine transform (DCT) [1–3] and vector quantization (VQ) [4–10] are two popular methods in image compression. The DCT approach has an excellent energy compaction property and requires only real operations in transformation process. DCT, defined by Ahamed et al. [1] in 1974, has been applied in many fields such as signal processing, data compression, filtering, and feature extraction in image processing. It is close to the optimal transform for the first-order Markov image in energy compaction and consequently, in decorrelating a signal.

Clustering or codebook design is an essential process in image compression based on vector quantization. Codebook design can be considered as a clustering process in which the training vectors are classified into the specific classes based on the minimization of average distortion between the training vectors and codebook vectors (classes' centers). The goal of VQ is to create a codebook for which the average distortion generated by approximating a training vector and a codeword in a codebook is minimized. Vector quantization is a significant methodology in image compression, in which blocks of divided pixels are formed as training vectors rather than individual scales. Such a method results in the massive reduction of the image information in image transmission. The image is reconstructed by replacing each image block with its nearest codevector.

The fuzzy clusters are generated by dividing the training samples in accordance with the membership function. The fuzzy c-means algorithms (FCM) use the probabilistic constraint to enable the memberships of a training sample across clusters that sum up to 1, which means the different grades of a training sample are shared by distinct clusters,

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but not as degrees of typicality. In contrast, each component generated by the possibilistic c-means (PCM) corresponds to a dense region in the data set. Each cluster is independent of the other clusters in the PCM strategy. Memberships and typicalities are both important for the correct feature of data substructure in clustering problem. If a training sample has been classified to a suitable cluster, then membership is a better constraint for which the training sample is closest to this cluster. In other words, typicality is an important factor for unburdening the undesirable effects of outliers to compute the cluster centers. The penalized term was added into fuzzy c-means by Yang [11] and Yang and Su [12] to construct the penalized fuzzy c-means (PFCM) algorithm. Lin [13] also embedded the compensated constraint into FCM to create compensated fuzzy c-means (CFCM) algorithm. The performances in clustering problem can be updated in PFCM and CFCM that have been proven by Yang [11] and Lin [13], respectively. In the proposed approaches, the problem of the vector quantization is regarded as a process of the minimization of a cost function. This cost function is defined as the average distortion between the training vectors in AC information transformed by DCT to the cluster centers represented by the codevectors in the codebook. The training vectors, constructed by AC information in DCT transformation, are directly fed into these unsupervised algorithms. Then, the cluster centroids are updated using membership and typicality functions with penalized or compensated constraints. However, a training vector does not necessarily belong to one class. Instead, a certain membership grade belonging to proper class is associated with every training vector, and typicality related to the mode of the cluster can also be calculated based on all n training samples. Consequently, the cluster centroids, membership grades, and typicality degrees can be updated to remove outliers and to speed up the energy converging into a near-global minimum in order to produce a satisfactory codebook. In a simulated study, the proposed approaches are described to have the capability for VQ in DCT for image compression, whose promising results are shown.

The remainder of this paper is organized as follows. Section 2 discusses VQ and DCT algorithms in image compression. FCM approaches are presented in Section 3. Possibilistic clustering technique is shown in Section 4; Section 5 proposes FPCM strategy; Section 6 presents penalized fuzzy possibilistic c-means (PFPCM) and compensated fuzzy possibilistic c-means (CFPCM) methods for VQ in DCT domain. Section 7 shows several experimental results; Finally, Section 8 gives the discussion and conclusions.

2. VQ and DCT

VQ is an important methodology in image compression, in which blocks of divided pixels are formed as training vectors. Such a method results in massive reduction of the image information in image transmission. The image is reconstructed by replacing each image block with its nearest codevector. The dimensions, with $N \times N$ pixels in an image, can be divided into n blocks (vectors of pixels) and each block occupies $\lambda \times \lambda$ ($\lambda < N$) pixels. A vector quantization is a technique that maps training vectors $\{\mathbf{X}_x,\ x=1,2,\ldots,n\}$ in Euclidean $\lambda \times \lambda$-dimensional space $R^{\lambda\times\lambda}$ into a set $\{\mathbf{Y}_x,\ x=1,2,\ldots,n\}$ of points in $R^{\lambda\times\lambda}$, called a codebook. The mapping is usually defined to minimize expected distortion measure, $E[d(\mathbf{X}_x, \mathbf{Y}_y)]$, using the mean square error (MSE) given by $d(\mathbf{x},\mathbf{y}) = (\mathbf{x}-\mathbf{y})^{\mathrm{T}}(\mathbf{x}-\mathbf{y})$.

Transform coding has been popularly used in image compression. Non-overlapping blocks are transformed to produce an array of coefficients. The idea is to remove the redundancy efficiently from image pixels in the transform domain. The DCT is one of commonly used transform-coding methods in image compression. The DCT has been shown to posses a superior energy compacting property. A frequency spectrum $F(r,s)$ of an $N \times N$ image represented by $f(j,k)$ for $j,k = 0,1,\ldots,N-1$ can be defined as

$$F(r,s) = \frac{4C(r)C(s)}{N^2} \sum_{j=0}^{N-1}\sum_{k=0}^{N-1} f(j,k) \times \cos\frac{(2j+1)r\pi}{2N} \cos\frac{(2k+1)s\pi}{2N}. \tag{1}$$

The inverse DCT (IDCT) is, therefore, defined as

$$f(j,k) = \sum_{j=0}^{N-1}\sum_{k=0}^{N-1} C(r)C(s)F(r,s) \times \cos\frac{(2j+1)r\pi}{2N} \cos\frac{(2k+1)s\pi}{2N}, \tag{2}$$

where the values of r and s are also from 0 to $N-1$ and C is defined as

$$C(w) = \begin{cases} \dfrac{1}{\sqrt{2}} & \text{for } w = 0, \\ 1 & \text{for } w = 1,2,\ldots,N-1. \end{cases} \tag{3}$$

The input image will be decomposed into non-overlapping blocks of equal size at the first stage in the system. After the image is blocked, the DCT is applied to transform each block into frequency spectrum of DC and AC terms. Then, the AC coefficients are recoded as 6-bit positive values (expressed values from -32 to 32) to be fed into the proposed algorithms for training vectors and generating better codebook off-line. Then, the DC term and the indexes searched from codebook with the minimum distortion are transmitted in the encoding process. In decoder, indexes of codevectors are received and recoded to get AC information (codevectors). Then, the IDCT is used to reconstruct the image using the received DC and the reconstructed AC information. The detail procedure for the encoder and decoder systems is shown in Fig. 1. One DC and 15 AC values in a 4×4 block are also shown in Fig. 2.

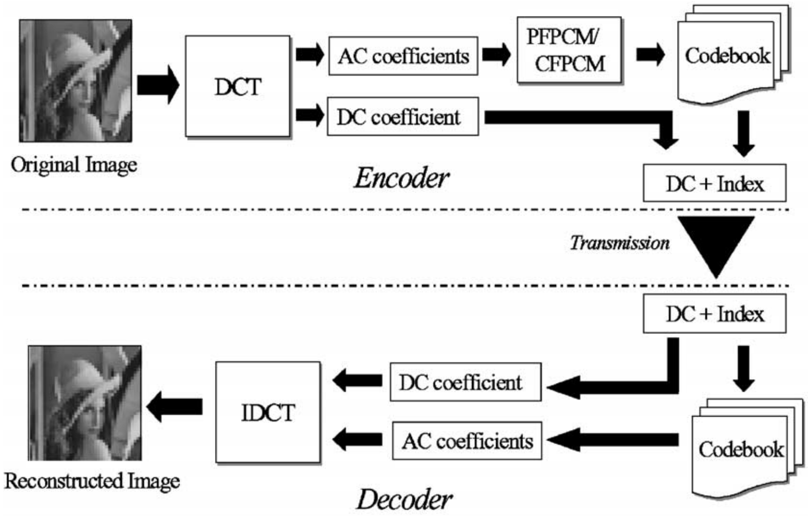

Fig. 1. The procedure of the encoder and decoder system.

DC	AC1	AC2	AC3
AC4	AC5	AC6	AC7
AC8	AC9	AC10	AC11
AC12	AC13	AC14	AC15

Fig. 2. DC and AC values in a 4×4 block.

3. Fuzzy clustering techniques

Fuzzy clustering strategies are mathematical tools for detecting similarities between the members of a collection of samples. The theory of fuzzy logic provides a mathematical framework to capture the uncertainties associated with the human cognition processes. Unlike the hard c-means method, in FCM every training sample belongs to every cluster with some degree of membership. In the following subsections, fuzzy c-means strategies are reviewed.

3.1. Fuzzy c-means algorithms

The fuzzy set theory has been applied in different fields since its introduction in 1965 by Zadeh [14]. The theory of fuzzy logic provides a mathematical environment to capture the uncertainties of the same human cognition processes. The FCM algorithms clustering strategy was first presented by Dunn [15], and an associated conception and strategy were proposed by Bezdek [16]. The purpose of the FCM approaches, like the conventional clustering techniques, is to group data into clusters of similar items by minimizing a least squared-error measure. For $c \geqslant 2$ (c is the number of clusters) and $m > 1$, the algorithm chooses $\mu_x : Z \to [0,1]$ so that $\sum_x \mu_x = 1$ and $\varpi_i \in R^d$ for $i = 1, 2, \ldots, c$ to minimize the objective function

$$J_{FCM} = \frac{1}{2} \sum_{x=1}^{n} \sum_{i=1}^{c} (\mu_{x,i})^m ||z_x - \varpi_i||^2, \tag{4}$$

where $\mu_{x,i}$ is the value of the ith membership grade on the xth sample z_x. The cluster centers $\varpi_1, \ldots, \varpi_j, \ldots, \varpi_c$ can be regarded as prototypes for the clusters represented by the membership grades. For the purpose of minimizing the objective function, the cluster centers and membership grades are chosen so that a high degree of membership occurs for samples closer to the corresponding cluster centers. The membership grades and cluster centers are iteratively updated by the following formulas:

$$\mu_{x,i} = \left(\sum_{\ell=1}^{c} \frac{(||z_x - \varpi_i||^2)^{1/(m-1)}}{(||z_x - \varpi_\ell||^2)^{1/(m-1)}} \right)^{-1},$$

$$x = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, c \tag{5}$$

and

$$\varpi_i = \frac{1}{\sum_{x=1}^{n} (\mu_{x,i})^m} \sum_{x=1}^{n} (\mu_{x,i})^m z_x. \tag{6}$$

The value $m \in (1, \infty)$ is the fuzzification parameter (or exponential weight). This parameter reduces the sensitivity of the class centers to noise in the data.

3.2. Penalized fuzzy c-means and compensated fuzzy c-means

A variant of the fuzzy clustering methods PFCM [11,12] and CFCM [13] algorithms, added penalty and compensated terms to the objective function in the past. The PFCM and CFCM objective functions are reviewed as follows:

$$J_{PFCM} = \frac{1}{2}\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}||z_x - \varpi_i||^2 - \frac{1}{2}v\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}\ln\alpha_i$$

$$= J_{FCM} - \frac{1}{2}v\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}\ln\alpha_i \qquad (7)$$

and

$$J_{CFCM} = \frac{1}{2}\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}||z_x - \varpi_i||^2 + \frac{1}{2}\tau\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}\tanh(\alpha)$$

$$= J_{FCM} + \frac{1}{2}\tau\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}\tanh(\alpha_i), \qquad (8)$$

where α_i is a proportional constant of class i, while v ($v \geqslant 0$) and τ ($\tau \geqslant 0$) are constants. When $v = 0$ and $\tau = 0$, J_{PFCM} and J_{CFCM} are equal to J_{FCM}. The penalty and compensated terms, $-\frac{1}{2}v\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}\ln\alpha_i$ and $+\frac{1}{2}\tau\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}\tanh(\alpha_i)$, are added to the objective function, α_i is defined as

$$\alpha_i = \frac{\sum_{x=1}^{n}\mu_{x,i}^{m}}{\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}}, \quad i = 1,2,\ldots,c \qquad (9)$$

and membership functions $\mu_{x,i}$ for PFCM and CFCM are shown as

$$(\mu_{x,i})_{PFCM} = \left(\sum_{\ell=1}^{c}\frac{(||z_x - \varpi_i||^2 - v\ln\alpha_i)^{1/(m-1)}}{(||z_x - \varpi_\ell||^2 - v\ln\alpha_\ell)^{1/(m-1)}}\right)^{-1},$$

$$\times x = 1,2,\ldots,n, \quad i = 1,2,\ldots,c \qquad (10)$$

and

$$(\mu_{x,i})_{CFCM} = \left(\sum_{\ell=1}^{c}\frac{(||z_x - \varpi_i||^2 + \tau\tanh(\alpha_i))^{1/(m-1)}}{(||z_x - \varpi_\ell||^2 + \tau\tanh(\alpha_\ell))^{1/(m-1)}}\right)^{-1},$$

$$x = 1,2,\ldots,n, \quad i = 1,2,\ldots,c, \qquad (11)$$

respectively. The fuzzy clustering methods not only suffer from the crisp strategies, but also from the presence of noise in the data. Most distance functions are geometric in nature, so that noise points are often distant from the predefined subregions and influence the resulting partition. In unsupervised techniques, it is a serious problem to generate membership functions from training data when the nature and numbers of clusters in the data set are unknown.

4. Possibilistic clustering technique

The theory of fuzzy logic provides a mathematical environment to capture the uncertainties in much the same human cognition processes. The fuzzy clusters are generated by dividing the training samples in accordance with the membership functions matrix $U = [\mu_{x,i}]$. The component $\mu_{x,i}$ denotes the grade of membership that a training sample belongs to a cluster. Real data unavoidably involves some noises, either from interface due to noise sources which exist in the natural environment or from the equipment itself. Therefore, the drawback of PCM will be significant while processing improper data. The purpose of the FCM approaches, like the conventional clustering techniques, is to group data into clusters of similar items by minimizing a least-squared error measure. The FCM algorithms use the probabilistic constraint to enable the memberships of a training sample across clusters to sum up to 1, which means the different grades of a training sample are shared by distinct clusters but not as degrees of typicality. In contrast, each component generated by the PCM corresponds to a dense region in the data set. Each cluster is independent of the other clusters in the PCM strategy. The PCM was proposed by Krishnapuram et al. [17,18] for unsupervised clustering. The objective function of the PCM can be formulated as

$$J_{PCM} = \frac{1}{2}\sum_{x=1}^{n}\sum_{i=1}^{c}(t_{x,i})^{\eta}||z_x - \varpi_i||^2 + \sum_{x=1}^{n}\gamma_i\sum_{i=1}^{c}(1 - t_{x,i})^{\eta}, \qquad (12)$$

where

$$\gamma_i = \frac{\sum_{x=1}^{n}t_{x,i}^{\eta}||z_x - \varpi_i||^2}{\sum_{x=1}^{n}t_{x,i}^{\eta}}$$

is the scale parameter at the ith cluster,

$$t_{x,i} = \frac{1}{1 + (||z_x - \varpi_i||^2/\gamma_i)^{1/(\eta-1)}}$$

is the possibilistic typicality value of training sample z_x belonging to the cluster i. $\eta \in [1,\infty)$ is a weighting factor called the possibilistic parameter. Typical of other cluster approaches, the PCM also depends on initialization. In PCM techniques, the clusters do not have a lot of mobility, since each data point is classified as only one cluster at a time rather than all the clusters simultaneously. Therefore, a suitable initialization is required for the algorithms to converge to nearly global minimum. Barni et al. [19] indicated that the price PCM pays for its freedom to ignore noisy points in that PCM is very sensitive to good initializations, and it sometimes generates coincident clusters.

5. Fuzzy-possibilistic c-means

Memberships and typicalities are both important for the correct feature of data substructure in clustering problem.

If a training sample has been classified into a suitable cluster, membership is a better constraint for which the training sample is closest to this cluster. On the other hand, typicality is an important factor for unburdening the undesirable effects of outliers to compute the cluster centers. In accordance with Ref. [20], typicality is related to the mode of the cluster and can be calculated based on all n training samples. Thus, an objective function in the FPCM depending on both memberships and typicalities can be shown as

$$J_{FPCM} = \frac{1}{2}\sum_{x=1}^{n}\sum_{i=1}^{c}(\mu_{x,i}^{m} + t_{x,i}^{\eta})||z_x - \varpi_i||^2, \tag{13}$$

where memberships, typicalities, and centroids are defined as

$$\mu_{x,i} = \left(\sum_{\ell=1}^{c}\frac{(||z_x - \varpi_i||^2)^{1/(m-1)}}{(||z_x - \varpi_\ell||^2)^{1/(m-1)}}\right)^{-1}, \quad x = 1,2,\ldots,n,\ \ i = 1,2,\ldots,c, \tag{14}$$

$$t_{x,i} = \left(\sum_{y=1}^{n}\frac{(||z_x - \varpi_i||^2)^{1/(\eta-1)}}{(||z_y - \varpi_i||^2)^{1/(\eta-1)}}\right)^{-1}, \quad y = 1,2,\ldots,n,\ \ i = 1,2,\ldots,c \tag{15}$$

and

$$\varpi_i = \frac{1}{\sum_{y=1}^{n}(\mu_{y,i}^{m} + t_{y,i}^{\eta})}\sum_{x=1}^{n}(\mu_{x,i}^{m} + t_{x,i}^{\eta})z_x \tag{16}$$

individually. The FPCM produces not only membership grades but also typicality degrees in which the objective function a convex function just as FCM, which has been proved to converge in mathematics.

6. Penalized fuzzy possibilistic c-means and compensated fuzzy possibilistic c-means

Typical of the definition of PFCM and CFCM, fuzzy possibilistic clustering methods can also embed penalized and compensated terms to the objective function to construct PFPCM and CFPCM algorithms. The objective functions of PFPCM and CFPCM are defined as follows:

$$\begin{aligned} J_{PFPCM} &= \frac{1}{2}\sum_{x=1}^{n}\sum_{i=1}^{c}(\mu_{x,i}^{m} + t_{x,i}^{\eta})||z_x - \varpi_i||^2 \\ &\quad - \frac{1}{2}v\sum_{x=1}^{n}\sum_{i=1}^{c}(\mu_{x,i}^{m}\ln\alpha_i + t_{x,i}^{\eta}\ln\beta_x) \\ &= J_{FPCM} - \frac{1}{2}v\sum_{x=1}^{n}\sum_{i=1}^{c}(\mu_{x,i}^{m}\ln\alpha_i + t_{x,i}^{\eta}\ln\beta_x) \end{aligned} \tag{17}$$

and

$$\begin{aligned} J_{CFPCM} &= \frac{1}{2}\sum_{x=1}^{n}\sum_{i=1}^{c}(\mu_{x,i}^{m} + t_{x,i}^{\eta})||z_x - \varpi_i||^2 \\ &\quad + \frac{1}{2}\tau\sum_{x=1}^{n}\sum_{i=1}^{c}(\mu_{x,i}^{m}\tanh\alpha_i + t_{x,i}^{\eta}\tanh\beta_x) \\ &= J_{FPCM} + \frac{1}{2}\tau\sum_{x=1}^{n}\sum_{i=1}^{c}(\mu_{x,i}^{m}\tanh\alpha_i + t_{x,i}^{\eta}\tanh\beta_x), \end{aligned} \tag{18}$$

where α_i is a proportional constant of class i, β_x is a proportional constant of training vector z_x, and v $(v \geqslant 0)$, τ $(\tau \geqslant 0)$ are also constants. When $v = 0$ and $\tau = 0$, both J_{PFPCM} and J_{CFPCM} are equal to J_{FPCM}. The penalized and compensated terms, $-\frac{1}{2}v\sum_{x=1}^{n}\sum_{i=1}^{c}(\mu_{x,i}^{m}\ln\alpha_i + t_{x,i}^{\eta}\ln\beta_x)$ and $\frac{1}{2}\tau\sum_{x=1}^{n}\sum_{i=1}^{c}(\mu_{x,i}^{m}\tanh\alpha_i + t_{x,i}^{\eta}\tanh\beta_x)$, are added to the objective functions J_{PFPCM} and J_{CFPCM}, respectively. In these functions, α_i and β_x are defined as

$$\alpha_i = \frac{\sum_{x=1}^{n}\mu_{x,i}^{m}}{\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}}, \quad i = 1,2,\ldots,c, \tag{19}$$

$$\beta_x = \frac{\sum_{i=1}^{c}t_{x,i}^{\eta}}{\sum_{x=1}^{n}\sum_{i=1}^{c}t_{x,i}^{\eta}}, \quad x = 1,2,\ldots,n. \tag{20}$$

Membership $\mu_{x,i}$ and typicality $t_{x,i}$ for PFPCM and CFPCM are, respectively, shown as

$$(\mu_{x,i})_{PFPCM} = \left(\sum_{\ell=1}^{c}\frac{(||z_x - \varpi_i||^2 - v\ln\alpha_i)^{1/(m-1)}}{(||z_x - \varpi_\ell||^2 - v\ln\alpha_\ell)^{1/(m-1)}}\right)^{-1}, \quad x = 1,2,\ldots,n,\ \ i = 1,2,\ldots,c, \tag{21}$$

$$(t_{x,i})_{PFPCM} = \left(\sum_{y=1}^{n}\frac{(||z_x - \varpi_i||^2 - v\ln(\beta_x))^{1/(\eta-1)}}{(||z_y - \varpi_i||^2 - v\ln(\beta_y))^{1/(\eta-1)}}\right)^{-1}, \quad x = 1,2,\ldots,n,\ \ i = 1,2,\ldots,c \tag{22}$$

and

$$(\mu_{x,i})_{CFPCM} = \left(\sum_{\ell=1}^{c}\frac{(||z_x - \varpi_i||^2 + \tau\tanh(\alpha_i))^{1/(m-1)}}{(||z_x - \varpi_\ell||^2 + \tau\tanh(\alpha_\ell))^{1/(m-1)}}\right)^{-1}, \quad x = 1,2,\ldots,n,\ \ i = 1,2,\ldots,c, \tag{23}$$

$$(t_{x,i})_{CFPCM} = \left(\sum_{y=1}^{n}\frac{(||z_x - \varpi_i||^2 + \tau\tanh(\beta_x))^{1/(\eta-1)}}{(||z_y - \varpi_i||^2 + \tau\tanh(\beta_y))^{1/(\eta-1)}}\right)^{-1}, \quad x = 1,2,\ldots,n,\ \ i = 1,2,\ldots,c. \tag{24}$$

The centroid of cluster i is calculated in the same way as the definition in Eq. (16) for PFPCM and CFPCM.

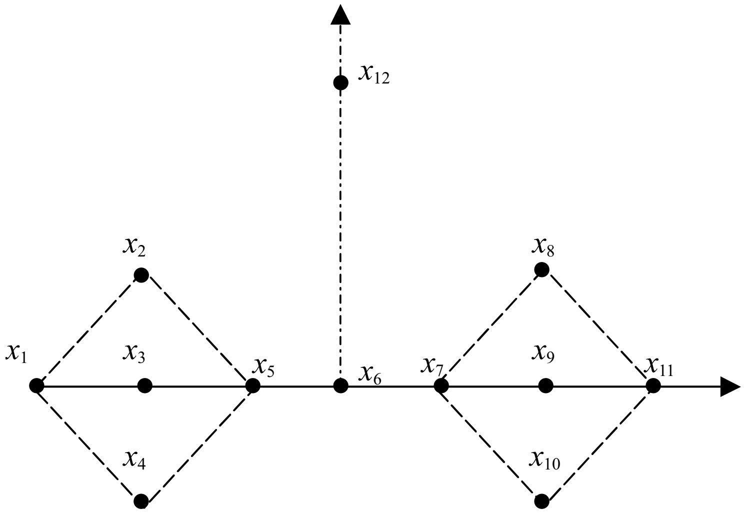

Fig. 3. Coordinates of the data set.

Table 1
The membership grades and typicality degrees for different methods with $m = 2$ and $\eta = 2$

	Data set		FPCM				PFPCM				CFPCM			
x	p1	p2	$\mu_{x,1}$	$\mu_{x,2}$	$t_{x,1}$	$t_{x,2}$	$\mu_{x,1}$	$\mu_{x,2}$	$t_{x,1}$	$t_{x,2}$	$\mu_{x,1}$	$\mu_{x,2}$	$t_{x,1}$	$t_{x,2}$
1	−5.00	0.00	0.003	0.997	0.0000	0.0032	0.0065	0.9935	0.0004	0.0550	0.0054	0.9946	0.0002	0.0397
2	−3.34	1.67	0.002	0.998	0.0000	0.0166	0.0046	0.9954	0.0008	0.1813	0.0034	0.9966	0.0005	0.1487
3	−3.34	0.00	0.000	1.000	0.0000	0.9665	0.0015	0.9985	0.0009	0.5701	0.0008	0.9992	0.0006	0.6649
4	−3.34	−1.67	0.009	0.991	0.0000	0.0026	0.0160	0.9840	0.0007	0.0448	0.0136	0.9864	0.0004	0.0320
5	−1.67	0.00	0.009	0.991	0.0001	0.0104	0.0210	0.9790	0.0028	0.1298	0.0165	0.9835	0.0017	0.1025
6	0.00	0.00	0.500	0.500	0.0005	0.0005	0.5001	0.4999	0.0134	0.0134	0.5000	0.5000	0.0086	0.0087
7	1.67	0.00	0.991	0.009	0.0107	0.0001	0.9790	0.0210	0.1298	0.0028	0.9835	0.0165	0.1023	0.0017
8	3.34	1.67	0.998	0.002	0.0173	0.0000	0.9954	0.0046	0.1807	0.0008	0.9966	0.0034	0.1478	0.0005
9	3.34	0.00	1.000	0.000	0.9653	0.0000	0.9985	0.0015	0.5706	0.0009	0.9992	0.0008	0.6659	0.0006
10	3.34	−1.67	0.991	0.009	0.0026	0.0000	0.9841	0.0159	0.0449	0.0007	0.9864	0.0136	0.0321	0.0004
11	5.00	0.00	0.997	0.003	0.0033	0.0000	0.9935	0.0065	0.0549	0.0004	0.9946	0.0054	0.0397	0.0002
12	0.00	10.00	0.500	0.500	0.0000	0.0000	0.4999	0.5001	0.0002	0.0002	0.4998	0.5002	0.0001	0.0001
Class center			(−3.0807, 0.3937) (3.0786, 0.3969)				(−3.0530, 0.4484) (3.0528, 0.4469)				(−3.0582, 0.4364) (3.0585, 0.4343)			

7. Experimental results

To compare the performance of FPCM, LBG, and the proposed approaches, a data set proposed by Pal et al. [20] and real images are used for simulation in an IBM Pentium III 166 MHz computer. The data set, shown in Fig. 3, consists of 12 points on a 2-D coordinate. Initially, the states of neurons $\mu_{x,i}$ and $t_{x,i}$ are randomly set during 0 to 1. Tables 1 and 2 show the final states of neurons $\mu_{x,i}$ and $t_{x,i}$ with $m = 2$, $\eta = 2$ and $m = 5$, $\eta = 2$, respectively. Table 3 shows the indices of the 12 points sorted by typicality values in each cluster. Points 1–5 are most typical to cluster 1 and points 7–11 are also most typical to cluster 2. Point 6 has equal typicality values in both clusters. Although point 12 also occupies equal typicality values in both clusters, it is an order of magnitude smaller than the typicality value for point 6, which means that point 6 belongs to both clusters with proper grades more strongly than point 12. This also means that the PFPCM and CFPCM can prune outliers from the data to reduce the effects of noise. Pal et al. indicated that the typicality parameter η might be best chosen in the interval 3–5 with initial centroids. This fact is proved in FPCM, PFPCM, and CFPCM, respectively, with random centroid values in this paper. With a small η ($\eta = 2, m = 2$),

Table 2
The membership grades and typicality degrees for different methods with $m = 5$ and $\eta = 2$

x	Data set p1	Data set p2	FPCM $\mu_{x,1}$	FPCM $\mu_{x,2}$	FPCM $t_{x,1}$	FPCM $t_{x,2}$	PFPCM $\mu_{x,1}$	PFPCM $\mu_{x,2}$	PFPCM $t_{x,1}$	PFPCM $t_{x,2}$	CFPCM $\mu_{x,1}$	CFPCM $\mu_{x,2}$	CFPCM $t_{x,1}$	CFPCM $t_{x,2}$
1	−5.00	0.00	0.1644	0.8356	0.0000	0.0000	0.1942	0.8058	0.0003	0.0749	0.1852	0.8148	0.0002	0.0493
2	−3.34	1.67	0.1842	0.8158	0.0000	0.0000	0.2135	0.7865	0.0006	0.1057	0.2047	0.7953	0.0003	0.0701
3	−3.34	0.00	0.0174	0.9826	0.0000	0.9999	0.1488	0.8512	0.0006	0.6714	0.1261	0.8739	0.0003	0.7822
4	−3.34	−1.67	0.2050	0.7950	0.0000	0.0000	0.2379	0.7621	0.0006	0.0572	0.2283	0.7717	0.0003	0.0384
5	−1.67	0.00	0.2519	0.7481	0.0000	0.0000	0.2849	0.7151	0.0019	0.0783	0.2756	0.7244	0.0010	0.0531
6	0.00	0.00	0.5001	0.4999	0.0000	0.0000	0.4995	0.5005	0.0085	0.0085	0.5000	0.5000	0.0048	0.0048
7	1.67	0.00	0.7483	0.2517	0.0000	0.0000	0.7140	0.2860	0.0798	0.0019	0.7244	0.2756	0.0531	0.0010
8	3.34	1.67	0.8158	0.1842	0.0000	0.0000	0.7862	0.2138	0.1075	0.0006	0.7953	0.2047	0.0701	0.0003
9	3.34	0.00	0.9826	0.0174	0.9999	0.0000	0.8502	0.1498	0.6670	0.0006	0.8739	0.1261	0.7822	0.0003
10	3.34	−1.67	0.7950	0.2050	0.0000	0.0000	0.7613	0.2387	0.0577	0.0005	0.7717	0.2283	0.0384	0.0002
11	5.00	0.00	0.8355	0.1645	0.0000	0.0000	0.8055	0.1945	0.0753	0.0005	0.8148	0.1852	0.0493	0.0002
12	0.00	10.00	0.5000	0.5000	0.0000	0.0000	0.5000	0.5000	0.0001	0.0001	0.5000	0.5000	0.0001	0.0001
Class center			(−3.3591, 0.1172) (3.3584, 0.1174)				(−3.3493, 0.1916) (3.3534, 0.1950)				(−3.3554, 0.1674) (3.3557, 0.1676)			

Table 3
The indices of the 12 points corresponding to a sort on $t_{x,1}$ and $t_{x,2}$ for different methods with $\eta = 2$ and distinct values of m

Normal sort		FPCM				PFPCM				CFPCM			
$t_{x,1}$	$t_{x,2}$	$m=5$		$m=2$		$m=5$		$m=2$		$m=5$		$m=2$	
		$t_{x,1}$	$t_{x,2}$	$t_{x,1}$	$t_{x,2}$	$t_{x,1}$	$t_{x,2}$	$t_{x,1}$	$t_{x,2}$	$t_{x,1}$	$t_{x,2}$	$t_{x,1}$	$t_{x,2}$
9	3	9	3	9	3	9	3	9	3	9	3	9	3
8	2	1, 2, 3,	1, 2, 4	8	2	8	2	8	2	8	2	8	2
7	5	4, 5, 6,	5, 6, 7,	7	5	7	5	7	5	7	5	7	5
11	1	8, 10,	8, 9,	11	1	11	1	11	1	11	1	11	1
10	4	11, 12	10, 11,	10	4	10	4	10	4	10	4	10	4
6	6		12	6	6	6	6	6	6	6	6	6	6
5	7			5	7	5	7	5	7	5	7	5	7
3	9			1, 2,	8, 9	2, 3, 4	8, 9	3	9	2, 3, 4	8, 9	3	9
2	8			3, 4,	10, 11	1	10, 11	2	8	1	10, 11	2	8
4	10			12	12	12	12	4	10	12	12	4	10
1	11							1	11			1	11
12	12							12	12			12	12

the proposed approaches PFPCM and CFPCM can always obtain the correct sort for the indices of the 12 points except for a big value of m ($\eta = 2, m = 5$). But the sorting results, shown in Table 3, are constantly confused with a small value of typicality parameter for any m in the FPCM.

In the application of VQ in DCT domain, the quality of the images reconstructed from the designed methods DCT + PFPCM (VQ) and DCT + CFPCM (VQ) was compared with DCT + FPCM (VQ), DCT + LBG (VQ), and conventional VQ method LBG, respectively. The training vectors were extracted from 256 × 256 real images with 8-bit gray levels, which were divided into 4 × 4 blocks to generate 4096 non-overlapping 16-dimensional vectors and transmitted to DCT block. Three codebooks of size 64, 128 and 256 were generated for simulation. The root mean-squared error (RMSE) and peak signal-to-noise ratio (PSNR), evaluated in the reconstructed image and original image, respectively, are defined as follows:

$$RMSE = \sqrt{\frac{1}{N^2}\sum_{x=1}^{N}\sum_{y=1}^{N}[f(x,y) - \hat{f}(x,y)]^2} \tag{25}$$

and

$$PSNR = 10\log_{10}\frac{255 \times 255}{RMSE^2}, \tag{26}$$

Fig. 4. LENA image and its reconstructed images with 64 codewords: (a) original image; (b) LBG; (c) DCT + LBG (VQ); and (d) DCT + CFPCM (VQ).

where $f(x,y)$, $\hat{f}(x,y)$, and 255 are original image, reconstructed image, and peak value of pixels in an image. Parameters of all experiments for the real image in this paper are set as $v=\tau=0.5$ and $m=\eta=3$, respectively. Fig. 4 shows the "LENNA" image and its reconstructed images using LBG, DCT + LBG (VQ), and DCT + CFPCM (VQ) algorithms with 64 codewords, respectively. Fig. 5 shows the "F-16" image and its reconstructed images using DCT+LBG (VQ), DCT+FPCM (VQ), and DCT+CFPCM (VQ) approaches with 256 codewords, respectively. The average PSNRs for all reconstructed images completed by the listed methods using DCT + VQ are higher by 2.3–3 dB than those done by conventional VQ method LBG.

Tables 4 and 5 show the PSNR and RMSE of the images "LENNA" and "F-16" reconstructed from the various methods with different codebook sizes. In accordance with Tables 4 and 5, better results can be obtained with the proposed DCT+PFPCM (VQ) and DCT+CFPCM (VQ) algorithms. In Table 6, the reconstruction performance of PSNRs for the CFPCM and PFPCM is better than the FPCM and LBG by about 0.3–0.5 dBs and 2–3 dB in frequency domain, respectively. In summary, from the experiment results, the proposed algorithms could satisfactorily produce the codebook design in the DCT domain, while convergence of the objective function is guaranteed.

8. Discussion and conclusions

FPCM embedded with penalized and compensated constraints named CFPCM and PFPCM for VQ in DCT domain have been presented to enhance the reconstruction performance and reduce the block effect in this paper. In the simulation of modified butterfly pattern, CFPCM, PFPCM, and FPCM can always prune outliers. But PFCM is more sensitive than the proposed CFPCM and PFPCM with a small typicality value η so that PFCM always confuses the typicality order of indices in the simulated pattern with a random initialization. From the experimental results, the proposed PFPCM and CFPCM algorithms in frequency domain pro-

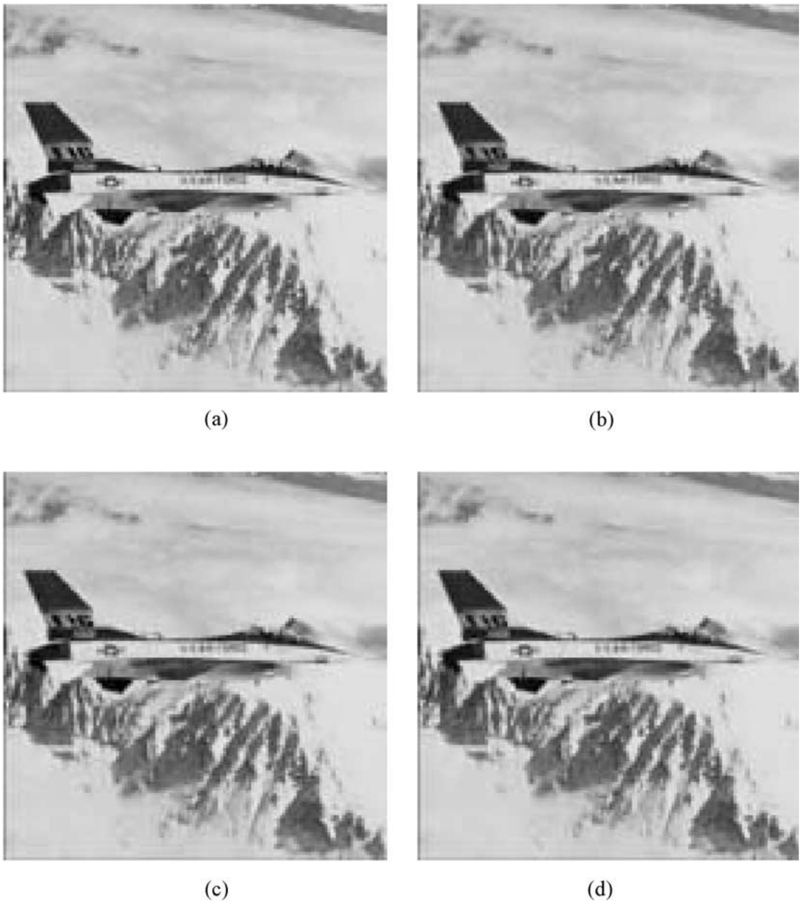

Fig. 5. F-16 image and its reconstructed images with 256 codewords: (a) original image; (b) DCT + LBG (VQ); (c) DCT + FPCM (VQ); and (d) DCT + CFPCM (VQ).

duces reconstructed images which are more promising than those reconstructed by the LBG (VQ), DCT + LBG (VQ), and DCT + FPCM (VQ) algorithms. The proposed strategies differ from the conventional FPCM, PFCM, and CFCM in which the possibilistic reasoning strategy is imposed on fuzzy clustering with penalized and compensated constraints for updating the grades of membership and typicality.

9. Summary

In this paper, two approaches called fuzzy possibilistic c-means (PFPCM) and compensated fuzzy possibilistic c-means (CFPCM) to vector quantization (VQ) in discrete cosine transform (DCT) for image compression are proposed. These two techniques embedded the penalized and compensated constraints into fuzzy possibilistic c-means (FPCM) to update the performance in reasoning process. The main purpose is to modify the FPCM strategy with penalized or compensated constraints so that the cluster centroids can be updated with penalized or compensated terms iteratively in order to find near-global solution in optimal problem. The information transformed by DCT was separated into DC and AC coefficients. Then, the AC coefficients are trained by using the proposed methods to generate a better codebook based on VQ. These two techniques have proven that the outlier noises can be more efficiently removed than the FPCM in the experimental results. From compression simulations, the promising performances can also be obtained using the proposed approaches than the FPCM and conventional VQ methods.

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Table 4
Reconstruction performance on LENA image under different algorithms

Algorithms	Codebook size	PSNR	RMSE
LBG (VQ)	64	25.26	12.12
	128	26.37	11.27
	256	27.06	10.33
DCT + LBG (VQ)	64	28.56	9.43
	128	29.05	8.85
	256	29.75	8.17
DCT + FPCM (VQ)	64	29.18	8.70
	128	30.05	7.98
	256	30.75	7.20
DCT + PFPCM (VQ)	64	29.25	8.63
	128	30.18	7.93
	256	30.88	7.14
DCT + CFPCM (VQ)	64	29.31	8.51
	128	30.28	7.83
	256	31.02	7.03

Table 5
Reconstruction performance on F-16 image under different algorithms

Algorithms	Codebook size	PSNR	RMSE
LBG (VQ)	64	24.11	14.91
	128	25.29	12.78
	256	26.34	11.53
DCT + LBG (VQ)	64	27.30	11.00
	128	27.84	10.35
	256	28.66	9.41
DCT + FPCM (VQ)	64	27.61	10.61
	128	28.54	9.54
	256	29.44	8.62
DCT + PFPCM (VQ)	64	27.69	10.53
	128	28.66	9.40
	256	29.43	8.60
DCT + CFPCM (VQ)	64	27.75	10.45
	128	28.79	9.32
	256	29.51	8.53

Table 6
PSNRs of images reconstructed from a codebook with size = 256 by using CFPCM, PFPCM, FPCM and LBG in DCT domain and LBG in spatial domain

Method Image	DCT+CFPCM (VQ)	DCT+PFPCM (VQ)	DCT+FPCM (VQ)	DCT+LBD (VQ)	LBG (VQ)
Barbara	31.88	31.73	31.50	29.54	26.93
Boy–girl	35.96	35.80	35.65	33.26	31.20
Girl	35.29	34.97	34.82	32.90	29.69
Pepper	32.72	32.59	32.31	29.66	26.78

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